

# Elliptic Curves and Modular Forms

## Mid Terminal Examination

September 13 2016

This exam is of 40 marks. Please read all the questions carefully and do not cheat. You are allowed to use

- *Silverman and Tate - Rational Points on Elliptic Curves.*
- *Ireland and Rosen - A Classical Introduction to Modern Number Theory.*

Please feel free to use whatever theorems you have learned in class after stating them clearly.

1. Consider the elliptic curve

$$E : y^2 = x^3 + 5x$$

a. Compute the group of torsion points in  $E(\mathbb{Q})$ . (10)

b. Compute the rank of  $E(\mathbb{Q})$  (15)

2. Let  $\chi$  be a non-trivial multiplicative character of  $\mathbb{F}_p$  and  $\rho$  the non-trivial character of order 2. Show that (10)

$$\sum_t \chi(1 - t^2) = J(\chi, \rho)$$

where  $J(\chi, \rho)$  is the Jacobi sum.

3. Compute the number of points of  $X^3 + Y^3 = Z^3$  over  $\mathbb{F}_{31}$ . (5)